# From Physical to Human Capital Accumulation: Inequality and the Process of Development

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**Abstract**

This research develops a growth theory that captures the endogenous replacement of physical capital accumulation by human capital accumulation as a prime engine of economic growth in the transition from the Industrial Revolution to modern growth. The proposed theory offers a unified account for the effect of income inequality on the growth process of the currently advanced economies during this transition. It argues that the replacement of physical capital accumulation by human capital accumulation as a prime engine of economic growth has changed the qualitative impact of inequality on the process of development. In the early stages of the Industrial Revolution, when physical capital accumulation was the prime source of economic growth, inequality enhanced the process of development by channeling resources towards individuals whose marginal propensity to save is higher. In the later stages of the transition to modern growth, as human capital emerged as a prime engine of economic growth, equality alleviated the adverse effect of credit constraints on human capital accumulation and promoted the growth process. As wages increase, however, credit constraints become less binding, differences in the marginal propensity to save decline and the aggregate effect of income distribution on the growth process becomes therefore less significant.

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# 1 Introduction

This research develops a growth theory that captures the endogenous replacement of physical capital accumulation by human capital accumulation as a prime engine of economic growth in the transition from the Industrial Revolution to modern growth. The proposed theory offers a unified account for the effect of income inequality on the growth process of the currently advanced economies during this transition. It argues that the replacement of physical capital accumulation by human capital accumulation as a prime engine of economic growth has changed the qualitative impact of inequality on the process of development. In the early stages of the Industrial Revolution, when physical capital accumulation was the prime source of economic growth, inequality enhanced the process of development by channeling resources towards individuals whose marginal propensity to save is higher. In the later stages of the transition to modern growth, as human capital emerged as a prime engine of economic growth, equality alleviated the adverse effect of credit constraints on human capital accumulation and promoted the growth process.

Existing theories regarding the effect of income distribution on the process of development can be classified into two categories distinguished by their conflicting predictions. The Classical approach suggests that inequality stimulates capital accumulation and thus promotes economic growth, whereas strands of the recent capital market imperfection approach suggest, in contrast, that for sufficiently wealthy economies equality stimulates investment in human capital and thus enhances the growth process.

The Classical approach was originated by Adam Smith (1776) and was further interpreted and developed by Keynes (1920), Lewis (1954), Kaldor (1957), and Bourguignon (1981). According to this approach, saving rates are an increasing function of wealth, and inequality therefore channels resources towards individuals whose marginal propensity to save is higher, increasing aggregate savings and capital accumulation and enhancing the process of development.

The Modern paradigm has been dominated by two complementary approaches. The capital market imperfection approach suggests that, in the presence of credit markets imperfection, equality in sufficiently wealthy economies alleviates the adverse effect of credit constraints on investment in human capital and thereby enhances economic growth (Galor and Zeira 1993).[1] The political economy approach proposes that equality diminishes the tendency for socio-political instability, or distortionary redistribution, and hence it stimulates investment and economic growth.[2]

[1]See Benabou (1996a, 2000), Durlauf (1996), and Fernandez and Rogerson (1996), as well. Perotti (1996) provides evidence in support of this link between eqaulity, human capital and growth. Alternatively, Banerjee and Newman (1993), Aghion and Bolton (1997), among others argued that equality positvely affects individual's investment opportunites that could be in physical rather than human capital. Consistently with the credit market imperfection approach, Easterly (2001) demonstrates the importantce of the middle class for the process of development.

[2]See the comprehensive survey of Benabou (1996b).

The proposed unified theory provides an intertemporal reconciliation between the conflicting viewpoints about the effect of inequality on economic growth. It suggests that the classical viewpoint, regarding the positive effect of inequality on the process of development, reflects the state of the world in early stages of industrialization when physical capital accumulation was the prime engine of economic growth. In contrast, the credit market imperfection approach regarding the positive effect of equality on economic growth reflects later stages of development when human capital accumulation becomes a prime engine of economic growth, and credit constraints are largely binding.[3] Hence, for the currently developed economies, the domination of the credit market imperfection channel in later stages of development is an inevitable by product of the domination of the classical mechanism in early stages of development.

The fundamental hypothesis of this research stems from the recognition that human capital accumulation and physical capital accumulation are fundamentally asymmetric. In contrast to physical capital, human capital is inherently embodied in humans and the existence of physiological constraints on brain capacity subjects its accumulation at the individual level to diminishing returns.[4] The aggregate return to investment in human capital is maximized therefore if the marginal returns are equalized across individuals. The aggregate stock of human capital would be therefore larger if its accumulation would be widely spread among individuals in society. This asymmetry between human and physical capital accumulation suggests therefore that as long as credit constraints are largely binding, equality is conducive for human capital accumulation, whereas provided that the marginal propensity to save increases with income, inequality is conducive for physical capital accumulation. Inequality therefore stimulates economic growth in stages of development in which physical capital accumulation is the prime engine of growth, whereas equality enhances economic growth in stages of development in which human capital accumulation is the dominating engine of economic growth and credit constraints are still largely binding.

The paper develops a growth model that captures the endogenous replacement of physical capital accumulation by human capital accumulation as a prime engine of economic growth in the transition of the currently advanced economies from the Industrial Revolution to modern growth. As argued by Abramovitch (1993 p.224) "In the nineteenth century, technological progress was heav-

[3]Fishman and Simhon (2002) analyze the effect of income distribution on economic growth in a model that combines the classical approach and the capital market imperfection approach. They argue that equality contributes to long-run growth in a monopolistically competitive economy only if individuals saving rates increase with income.

[4]One can argue that, although disembodied in humans, the accumulation of physical capital at the individuals level is also subjected to diminishing returns, due to agency problems resulting from asymmetric information, moral hazard, monitoring cost, etc. Nevertheless, since human capital is embodied in humans, it is apparent that the severity of diminishing return is larger in the context of human capital. Although the model is based on the simplifying assumption of constant return to physical capital at the individual level, the hypothesis developed in this paper is valid as long as the return to human capital accumulation at the individuals level diminishes faster than the return on physical capital.

ily biased in a physical capital-using direction. ... In the twentieth century, however, the physical capital-using bias weakened; it may have disappeared altogether. The bias shifted in an intangible (human and knowledge) capital-using direction and produced the substantial contribution of education and other intangible capital accumulation to this century productivity growth...". Indeed, evidence provided by Goldin and Katz (2001) and Abramovitz and David (2000) suggest that over the period 1890-1999 in the United States the contribution of human capital accumulation to the growth process has nearly doubled whereas the contribution of physical capital has declined significantly.[5]

The process of industrialization in England is characterized by a gradual increase in the relative importance of human capital accumulation, as well. It suggests that in the first phase of the Industrial Revolution (1760-1830), capital accumulation as a fraction of GNP has increased significantly whereas literacy rates remained largely unchanged. Skills and literacy requirements had been minimal and the state devoted virtually no resources to raise the level of literacy of the masses. Workers developed skills primarily through on-the-job training, and child labor was highly valuable. Consequently, literacy rates had not increased during the period 1750-1830 (Sanderson, 1995, pp. 2-10). The requirements for technical skills in that period, were slight and adequately met by traditional means (Green, 1990, pp. 293-294). As argued by Landes (1969, p 340) "although certain workers - supervisory and office personal in particular - must be able to read and do the elementary arithmetical operations in order to perform their duties, large share of the work of industry can be performed by illiterates as indeed it was especially in the early days of the industrial revolution."[6] In the second phase of the industrial revolution, however, capital accumulation subsided, the education of the labor force markedly increased and skills became necessary for production.[7] Investment ratio has increased from 6 percent in 1760 to 11.7 percent in the year 1831 and it remained around 11% on average in the period 1856-1913 (Crafts 1985, p. 73 and Matthews et al 1982, p. 137).[8] In contrast,

[5] Goldin and Katz (2001) show that the rate of growth of educational productivity was 0.29% per year over the period 1890-1915, accounting for about 11% of the 1.8% annual growth rate of output per capita over this period. In the period 1915-1999, the rate of growth of educational productivity was 0.53% per year accounting for about 20% of the 1.8% annual growth rate of output per capita over this period. (The labor share is assumed to be 0.7 over the entire period.) Abramovitz and David (2000) report that the fraction of the growth rate of output per capita that is directly attributed to physical capital accumulation has declined from an average of 56% in the period 1800-1890 to 31% in 1890-1927 and 21% in the period 1929-1966. Similarly, Denison (1962, p 270) suggests that the contribution of capital accumulation accounted for 22% of the growth rate in output per capita in the period 1909 - 1929 and 9% in the period 1929-1957, whereas the contribution of human capital accounted for 15% and 21%, respectively.

[6] Furthermore, as argued by Mitch (1992 pp. 14-15), during the first stages of the Industrial Revolution, literacy was largely a cultural skill or a hierarchy symbol that had limited value in the labor market. For instance, in 1841 only 4.9% of male workers and only 2.2% of female workers were in occupations in which literacy was strictly required. In addition, some have argued that the low skill requirements have even declined over this period. For instance, Sanderson (1972, p. 89) suggests that "One thus finds the interesting situation of an emerging economy creating a whole range of new occupations which require even less literacy and education than the old ones."

[7] From the 1850s, job advertisements suggest that literacy has become an increasingly desired characteristic for employment (Micth, 1993, p. 292).

[8] The emergence of human capital as a prime engine of economic growth in the second phase of the Industrial

the average years of schooling of the male labor force of England which has not changed significantly until 1830s, has tripled until the beginning of the twentieth century [Matthews et al (1982), p 573] and school enrollment of 10-year old has increased from 40% in 1870 to 100% in 1900.

The proposed model captures the asymmetry between the accumulation of human capital and physical capital accumulation and the role that this asymmetry plays in the determination of the effect of inequality on the growth process.[9] The model is based on three central elements in addition to the fundamental asymmetry between human capital and physical capital. First, the marginal propensity to save and to bequeath increases with wealth.[10] Inequality has therefore a positive effect on aggregate saving. This element captures the central mechanism in the classical approach.[11] Second, the economy is characterized by credit constraints that limit individual's borrowing. This element captures the central mechanism of the credit market imperfection approach. Credit constraint along with the inherent diminishing marginal returns in the production of human capital results in an inefficient investment in human capital.[12] Third, the economy is characterized by capital-skill complementarity. The accumulation of physical capital in early stages of development increases the potential rate of return to human capital and induces human capital accumulation.[13]

In early stages of industrialization physical capital is scarce, the rate of return to human capital is lower than the rate of return to physical capital and the process of development is fueled by capital accumulation. The positive effect of inequality on aggregate saving dominates therefore the negative effect on investment in human capital and inequality raises aggregate savings and capital

Revolution, channeled resources towards investment in human capital as well as investment in physical capital. Consequently, although aggregate investment in human and physical capital had increased, measured saving rates (where national accounts consider investment in education as expenditure) remained constant.

[9] Existing models that study the effect of wealth distribution on economic growth are based, in contrast, on either human capital or physical capital accumulation. This separation, as will be argued below, prevented these models from generating some of the fundamental insights derived in this paper.

[10] Dynan, Skinner and Zeldes (2000) find a strong positive relationship between personal saving rates and lifetime income in the United States. They find that saving rates rise from 3 percent in the lowest quintile to 25 percent in the top quintile, 44 percent in the top 5 percent and nearly 50 percent in the top 1 percent of the income distribution. They estimate the increase in saving rates between 1.5 percentage points and 3 percentage points for each $10,000 increase in annual permanent income. They argue that their findings are consistent with models in which precautionary saving and bequest motives drive variations in saving rates across income groups. Furthermore, Tomes (1981) and Menchik and David (1983) find evidence that the marginal propensity to bequeath increases with wealth.

[11] Indeed, investment ratio in the UK has nearly doubled during the period 1760-1831, as income per capita as well as income inequality grew in the course of the first phase of the Industrial Revolution. Some cross-country studies based on recent evidence find a positive impact of income inequality on total saving (e.g., Cook 1995 and Douglas 2001) and others do not find any significant effect (e.g., Schmidt-Hebbel and Serven 2000).

[12] As argued by Goldin and Katz (2001): "In Europe... the provision of formal secondary school education was - for most of the first half of the 20th century - generally limited to a small cadre of youth whose families could afford the private expanse or who had scored sufficiently well on an examination taken around age 11. Furthermore, cross section evidence shows an adverse effect of credit markets imperfection in the presence of inequality on human capital investment (e.g., Flug et. al. 1998 and Checchi 2001).

[13] The phenomena is documented empirically by Goldin and Katz (1998) for the United States in the 20th century. The European experience in the second half of the 19th century is consistent with this fundamental hypothesis as well. Evidence surveyed by Galor and Moav (2000b) suggests that in the second phase of the Industrial Revolution, education reforms in Europe were designed primarily to satisfy the increasing skill requirements in the process of industrialization.

accumulation and enhances the process of development. In later stages of development, as physical capital accumulates, the complementarity between capital and skills increases the rate of return to human capital so as to induce human capital accumulation, and the accumulation of human capital as well as physical capital fuel the process of development.[14] Since human capital is embodied in individuals and individual's investment in human capital is subjected to diminishing marginal returns, the aggregate return to investment in human capital is maximized if the marginal returns are equalized across individuals. Equality alleviates the adverse effect of credit constraints, and has therefore a positive effect on the aggregate level of human capital and economic growth. Moreover, the differences in the marginal propensities to save across individuals narrow as wages increase, and the negative effect of equality on aggregate saving subsides.[15] In later stages of development, therefore, as long as credit constraints are sufficiently binding, the positive effect of inequality on aggregate saving is dominated by the negative effect on investment in human capital and equality stimulates economic growth. As wages further increase, however, credit constraints become less binding, differences in the marginal propensity to save further decline, and the aggregate effect of income distribution on the growth process becomes less significant.[16]

The ordering of regimes is important for the understanding of the role of inequality in the process of development of the currently developed economies. Nevertheless, the insights that the effect of inequality is determined by the return to human capital relative to the return to physical - a unique feature of the proposed unified approach - is relevant for the currently LDCs as well. In contrast to the historical growth path of the currently developed economies, human capital accumulation may be the prime engine of economic growth in LDCs even in early stages of development due to the importation of capital and skilled-biased technologies.

The proposed unified theory generates a testable implication about the effect of inequality on economic growth that may provide a greatly needed theoretical guidance to resolve the empirical controversy about this relationship.[17] Unlike the previous theories the research suggests that the effect of inequality on growth depends on the relative return to physical and human capital. Inequality is beneficial for economic growth in economies in which the return to human capital relative to

[14] Although physical capital accumulation has increased the demand for human capital and therefore its potential rate of return, the supply response generated by individual's investment in education offset this potential increase. Furthermore, institutional changes may bring about an increase in supply that may even reduce the reward to human capital. For instance, the decline in the reward for education in the United States from the 1910th trough the 1940th despite a rapid skill-biased technological change is due to the growth of the relative supply of more educated labor that accelerated with the high school movement (Goldin and Katz 1998, 1999).

[15] Real wages had a positive trend in the UK since the beginning of the Industrial Revolution in 1750 until today (Harley 1993). Similarly, Margo (2000) reports that there was an upward trend in real earnings of non-farm workers in the US from 1860 to 1900 .

[16] Inequality may widen once again due to skilled or ability-biased technological change induced by human capital accumulation. This line of research was explored theoretically by Galor and Tsiddon (1997), Acemoglu (1998) and Galor and Moav (2000a), among others. It is supported empirically by Autor et al. (1999).

[17] See Banerjee and Duflo (2000), Barro (2000), Forbes (2000), Perotti (1996) and Quah (2002).

the return to physical capital is low, whereas equality is beneficial for economic growth in economies in which the relative return to human capital is high and credit constraints are largely binding. In contrast, the credit markets imperfection approach suggests, that the effect on inequality depends on the country's level of income - inequality is beneficial for poor economies and harmful for rich ones.

# 2 The Basic Structure of the Model

Consider an overlapping-generations economy in a process of development. In every period the economy produces a single homogeneous good that can be used for consumption and investment. The good is produced using physical capital and human capital. Output per-capita grows over time due to the accumulation of these factors of production. The stock of physical capital in every period is the output produced in the preceding period net of consumption and human capital investment, whereas the level of human capital in every period is the outcome of individuals' education decisions in the preceding period, subject to borrowing constraints.

## 2.1 Production of Final Output

Production occurs within a period according to a neoclassical, constant-returns-to-scale, production technology. The output produced at time $t$, $Y_t$, is

$$Y_t = F(K_t, H_t) \equiv H_t f(k_t) = AH_t k_t^{\alpha}; \qquad k_t \equiv K_t/H_t; \qquad \alpha \in (0,1), \tag{1}$$

where $K_t$ and $H_t$ are the quantities of physical capital and human capital (measured in efficiency units) employed in production at time $t$, and $A$ is the level of technology.[18] The production function, $f(k_t)$, is therefore strictly monotonic increasing, strictly concave satisfying the neoclassical boundary conditions that assure the existence of an interior solution to the producers' profit-maximization problem.

Producers operate in a perfectly competitive environment. Given the wage rate per efficiency unit of labor, $w_t$, and the rate of return to capital, $r_t$ , producers in period $t$ choose the level of employment of capital, $K_t$, and the efficiency units of labor, $H_t$, so as to maximize profits. That is, $\{K_t, H_t\} = \arg\max\,[H_t f(k_t) - w_t H_t - r_t K_t]$. The producers' inverse demand for factors of production is therefore

$$\begin{aligned} r_t &= f'(k_t) = \alpha A k_t^{\alpha-1} \equiv r(k_t); \\ w_t &= f(k_t) - f'(k_t)k_t = (1-\alpha)Ak_t^{\alpha} \equiv w(k_t). \end{aligned} \tag{2}$$

[18] For simplicity, the model abstracts from technological change. As discussed in the Concluding Remarks, the introduction of endogenous technological change does not affect the qualitative results.

## 2.2 Individuals

In every period a generation which consists of a continuum of individuals of measure 1 is born. Each individual has a single parent and a single child.[19] Individuals, within as well as across generations, are identical in their preferences and innate abilities. They may differ, however, in their family wealth and thus, due to borrowing constraints, in their investment in human capital.

Individuals live for two periods. In the first period of their lives individuals devote their entire time to the acquisition of human capital. The acquired level of human capital increases if their time investment is supplemented with capital investment in education.[20] In the second period of their lives (adulthood), individuals supply their efficiency units of labor and allocate the resulting wage income, along with their inheritance, between consumption and transfers to their children. The resources devoted to transfers are allocated between an immediate finance of their offspring's expenditure on education and saving for the future wealth of their offspring.

### 2.2.1 Wealth and Preferences

In the second period life, an individual $i$ born in period $t$ (a member $i$ of generation $t$) supplies the acquired efficiency units of labor, $h^i_{t+1}$, at the competitive market wage, $w_{t+1}$. In addition, the individual receives an inheritance of $x^i_{t+1}$. The individual's second period wealth, $I^i_{t+1}$, is therefore

$$I^i_{t+1} = w_{t+1}h^i_{t+1} + x^i_{t+1}. \tag{3}$$

The individual allocates this wealth between consumption, $c^i_{t+1}$, and transfers to the offspring, $b^i_{t+1}$. That is,

$$c^i_{t+1} + b^i_{t+1} \leq I^i_{t+1}. \tag{4}$$

The transfer of a member $i$ of generation $t$, $b^i_{t+1}$, is allocated between an immediate finance of their offspring's expenditure on education, $e^i_{t+1}$, and saving, $s^i_{t+1}$, for the future wealth of their offspring.[21] That is, the saving of a member $i$ of generation $t$, $s^i_{t+1}$, is

$$s^i_{t+1} = b^i_{t+1} - e^i_{t+1}. \tag{5}$$

The inheritance of a member $i$ of generation $t$, $x^i_{t+1}$, is therefore the return on the parental saving, $s^i_t$.

$$x^i_{t+1} = s^i_t R_{t+1} = (b^i_t - e^i_t)R_{t+1} \tag{6}$$

[19] As discussed in the Concluding Remarks, a more realistic family structure, based upon endogenous marriages and fertility decisions, would enrich the micro-foundations but would not affect the qualitative results.

[20] If alternatively, the time investment in education (foregone earnings) is the prime factor in the production of human capital, the qualitative results would not be affected, as long as physical capital would be needed in order to finance consumption over the education period. Both formulations assure that in the presence of capital markets imperfections investment in human capital depends upon family wealth.

[21] Parents finance the education of their offspring directly, subtracting the cost from the total intended bequest. This formulation of the saving function is consistent with the view that bequest as a saving motive is perhaps more important than life cycle considerations (e.g., Deaton 1992).

where $R_{t+1} \equiv 1 + r_{t+1} - \delta \equiv R(k_{t+1})$. For simplicity the rate of capital depreciation $\delta = 1$.[22]

Preferences of a member $i$ of generation $t$ are defined over consumption during adulthood,[23] $c^i_{t+1}$, and the value in period $t+1$ of total transfer to their offspring, $b^i_{t+1}$ (i.e., the sum of the immediate finance of the offspring's investment in human capital, $e^i_{t+1}$, and the saving for the offspring's future wealth, $s^i_{t+1}$). They are represented by a log-linear utility function that as will become apparent captures the spirit of Kaldorian-Keynesian saving behavior (i.e., the saving rate is an increasing function of wealth),[24]

$$u^i_t = (1-\beta)\log c^i_{t+1} + \beta \log(\overline{\theta} + b^i_{t+1}), \tag{7}$$

where $\beta \in (0,1)$ and $\overline{\theta} > 0$.[25]

### 2.2.2 The Formation of Human Capital

In the first period of their lives individuals devote their entire time for the acquisition of human capital. The acquired level of human capital increases if their time investment is supplemented with capital investment in education. However, even in the absence of real expenditure individuals acquire one efficiency unit of labor - basic skills. The number of efficiency units of labor of a member $i$ of generation $t$ in period $t+1$, $h^i_{t+1}$, is a strictly increasing, strictly concave function of the individual's real expenditure on education in period $t$, $e^i_t$.[26]

$$h^i_{t+1} = h(e^i_t), \tag{8}$$

where $h(0) = 1$, $\lim_{e^i_t \to 0^+} h'(e^i_t) = \gamma < \infty$, and $\lim_{e^i_t \to \infty} h'(e^i_t) = 0$. As is the case for the production of physical capital (which converts one unit of output into one unit of capital), the slope of the production function of human capital is finite at the origin. This assumption along with the ability of individuals to supply some minimal level of labor, $h(0)$, regardless of the physical investment in human capital (beyond time), assure that under some market conditions (non-basic) investment in

[22] $\delta \in [0,1]$ would not alter any of the qualitative results.

[23] The consumption of the child may be viewed as part of the consumption of the parent.

[24] Unlike Kaldor (1957) who assumes that the capitalists and workers differ in their saving behavior, the current formulation suggests that individuals are ex-ante identical in their intertemporal preferences although due to differences in income their marginal propensity to save may differ. Moav (2002) shows that long-run inequality could persists in Galor-Zeira (1993)'s framework, if this type of a "Keynesian saving function" replaces the assumption of non-convexities in the production of human capital.

[25] This form of altruistic bequest motive (i.e., the "joy of giving") is the common form in the recent literature on income distribution and growth. It is supported empirically by Altonji, Hayashi and Kotlikoff (1997) and Wilhelm (1996).

[26] A more realistic formulation would link the cost of education to (teacher's) wages, which may vary in the process of development. For instance, $h^i_{t+1} = h(e^i_t/w_t)$ implies that the cost of education is a function of the number of efficiency units of teachers that are used in the education of individual $i$. As will become apparent from (10) and (11), under both formulation the optimal capital expenditure on education, $e^i_t$, is an increasing function of the capital-labor ratio in the economy, and the qualitative results are therefore identical under both formulations.

human capital is not optimal.[27] The asymmetry between the accumulation of physical and human capital that is postulated in the paper is manifested in the larger degree of diminishing marginal productivity in the production of human capital (i.e., the strict concavity of $h(e_t^i)$ in contrast to the linearity of the production function of physical capital)

Given that the indirect utility function is a strictly increasing function of the individual's second period wealth, the unconstrained optimal real expenditure on education in every period $t$, $e_t^i$, from the viewpoint of individual $i$ of generation $t$, maximize the second period wealth, $I_{t+1}^i$.[28]

$$e_t^i = \arg\max[w_{t+1}h(e_t^i) + (b_t^i - e_t^i)R_{t+1}]. \tag{9}$$

Hence, as follows from the properties of $h(e_t^i)$, the optimal *unconstrained* real expenditure on education in every period $t$, $e_t$, is unique and identical across members of generation $t$.

If $R_{t+1} > w_{t+1}\gamma$ then $e_t = 0$, otherwise $e_t$ is given by

$$w_{t+1}h'(e_t) = R_{t+1}. \tag{10}$$

Moreover, since $w_{t+1} = w(k_{t+1})$ and $R_{t+1} = R(k_{t+1})$, it follows that $e_t = e(k_{t+1})$.

Given the properties of $f(k_t)$, there exists a unique capital-labor ratio $\widetilde{k}$, below which individuals do not invest in human capital (i.e., do not acquire non-basic skills). That is, $R(\widetilde{k}) = w(\widetilde{k})\gamma$, where $\lim_{e_t^i \to 0^+} h'(e_t^i) = \gamma$. As follows from (2), $\widetilde{k} = \alpha/(1-\alpha)\gamma \equiv \widetilde{k}(\gamma) > 0$ where $\widetilde{k}'(\gamma) < 0$. Since $R'(k_{t+1}) < 0$, $w'(k_{t+1}) > 0$, and $h''(e_t) < 0$, it follows that the optimal *unconstrained* real expenditure on education in every period $t$ is a function of the capital labor ratio in the subsequent period. In particualr,

$$e_t = e(k_{t+1}) \begin{cases} = 0 & if \quad k_{t+1} \leq \widetilde{k} \\ > 0 & if \quad k_{t+1} > \widetilde{k}, \end{cases} \tag{11}$$

where $e'(k_{t+1}) > 0$ for $k_{t+1} > \widetilde{k}$. Hence, if the capital-labor ratio in the next period is expected to be below $\widetilde{k}$ individuals do not acquire non-basic skills.

Suppose that individuals can not borrow.[29] It follows that the expenditure on education of a member $i$ of generation $t$, $e_t^i$ is limited by the aggregate transfer, $b_t^i$, that the individual receives. As follows from (10) and the strict concavity of $h(e_t)$, $e_t^i = b_t^i$ if $b_t^i \leq e_t$, whereas $e_t^i = e_t$ if $b_t^i > e_t$. That is, the expenditure on education of a member $i$ of generation $t$, $e_t^i$, is

$$e_t^i = \min[e(k_{t+1}), b_t^i]. \tag{12}$$

where $e_t^i$ is a non-decreasing function of $k_{t+1}$ and $b_t^i$.

[27] The Inada conditions are typically designed to simplify the exposition by avoiding corner solution, but they are surely not realistic assumptions.

[28] Once the aggregate intended transfers to the offspring is determined, the allocation of funds to the offspring education is assumed to optimal.

[29] Alternative specifications of capital markets imperfections e.g., finite differences between the interest rates for borrowers and lenders, would not affect the qualitative results.

### 2.2.3 Optimal Consumption and Transfers

A member $i$ of generation $t$ chooses the level of second period consumption, $c^i_{t+1}$, and a non-negative aggregate level of transfers to the offspring, $b^i_{t+1}$, so as to maximize the utility function subject to the second period budget constraint (4).[30]

Hence the optimal transfer of a member $i$ of generation $t$ is:

$$b^i_{t+1} = b(I^i_{t+1}) \equiv \begin{cases} \beta(I^i_{t+1} - \theta) & if \quad I^i_{t+1} \geq \theta; \\ 0 & if \quad I^i_{t+1} \leq \theta, \end{cases} \tag{13}$$

where $\theta \equiv \overline{\theta}(1-\beta)/\beta$. As follows from (13), the transfer rate $b^i_{t+1}/I^i_{t+1}$ is increasing in $I^i_{t+1}$. Moreover, as follows from (5) and (11) the saving of a member $i$ of generation $t-1$, $s^i_t$, is

$$s^i_t = \begin{cases} b^i_t & if \quad k_{t+1} \leq \widetilde{k}; \\ b^i_t - e^i_t & if \quad k_{t+1} > \widetilde{k}. \end{cases} \tag{14}$$

Hence, since $b^i_{t+1}/I^i_{t+1}$ is increasing in $I^i_{t+1}$, it follows from (12) that $s^i_{t+1}/I^i_{t+1}$ is increasing in $I^i_{t+1}$ as well. The transfer function and the implied saving function capture the properties of the Kaldorian-Keynesian saving hypothesis.

## 2.3 Aggregate Physical and Human Capital

Suppose that in period 0 the economy consists of two groups of adult individuals - Capitalists and Workers. They are identical in their preferences and differ only in their initial capital ownership. The Capitalists, denoted by $R$ (Rich), are a fraction $\lambda$ of all adult individuals in society, who equally own the entire *initial* physical capital stock. The Workers, denoted by $P$ (Poor), are a fraction $1-\lambda$ of all adult individuals in society, who have no ownership over the *initial* physical capital stock.[31] Since individuals are ex-ante homogenous *within* a group, the uniqueness of the solution to their optimization problem assures that their offspring are homogenous as well. Hence, in every period a fraction $\lambda$ of all adults are homogenous descendents of the Capitalist, denoted by members of group $R$, and a fraction $1-\lambda$ are homogenous descendents of Workers, denoted by members of group $P$.

The optimization of groups $P$ and $R$ of generations $t-1$ and $t$ in period $t$, determines the levels of physical capital, $K_{t+1}$, and human capital, $H_{t+1}$, in period $t+1$,

$$K_{t+1} = \lambda s^R_t + (1-\lambda)s^P_t = \lambda(b^R_t - e^R_t) + (1-\lambda)(b^P_t - e^P_t), \tag{15}$$

where $K_0 > 0$.

$$H_{t+1} = \lambda h(e^R_t) + (1-\lambda)h(e^P_t), \tag{16}$$

[30] It should be noted that the transfer, $b^i_{t+1}$, is necessarily non-negative due to the assumption that the offspring has no income in the first period of life.

[31] As will become apparent this class distinction will dissipate over time. In particular, the descendents of the working class will ultimately own some physical capital.

where in period 0 there is no (non-basic) human capital, i.e., $h_0^i = 1$ for all $i = R, P$ and thus $H_0 = 1$.[32]

Hence, (12) implies that the levels of physical capital, $K_{t+1}$, and human capital, $H_{t+1}$, in period $t+1$, are functions of intergenerational transfers in each of the groups, $b_t^R$ and $b_t^P$, and the capital labor ratio in the subsequent period, $k_{t+1}$.

$$\begin{aligned} H_{t+1} &= H(b_t^R, b_t^P, k_{t+1}); \\ K_{t+1} &= K(b_t^R, b_t^P, k_{t+1}). \end{aligned} \tag{17}$$

where (11), (12) and $e'(k_{t+1}) \geq 0$, imply that $\partial H_{t+1}/\partial k_{t+1} \geq 0$, $\partial K_{t+1}/\partial k_{t+1} \leq 0$, $H(b_t^R, b_t^P, 0) = 1$, and $K(b_t^R, b_t^P, 0) > 0$ for $b_t^R > 0$.

The capital-labor ratio in period $t+1$ is therefore,

$$k_{t+1} = \frac{K(b_t^R, b_t^P, k_{t+1})}{H(b_t^R, b_t^P, k_{t+1})}, \tag{18}$$

where the initial level of the capital labor ratio, $k_0$, is assumed to be

$$k_0 \in (0, \widetilde{k}). \tag{A1}$$

This assumption assures that in the initial stages the rate of return to physical capital is higher than the rate of return to human capital.

As follows from (11), this assumption is consistent with the assumption that the initial level of human capital is $H_0 = 1$.

Hence, it follows from (18) and the properties of the functions in (17) that there exists a continuous single valued function $\kappa(b_t^R, b_t^P)$ such that the capital-labor ratio in period $t+1$ is fully determined by the level of transfer of groups $R$ and $P$ in period $t$.

$$k_{t+1} = \kappa(b_t^R, b_t^P), \tag{19}$$

where $\kappa(0,0) = 0$ (since in the absence of transfers and hence savings the capital stock in the subsequent period is zero).

### 2.4 The Evolution of Transfers Within Dynasties

The evolution of transfers within each group $i = R, P$, as follows from (13), is

$$b_{t+1}^i = \max\{\beta[w_{t+1}h(e_t^i) + (b_t^i - e_t^i)R_{t+1} - \theta], 0\}; \qquad i = R, P. \tag{20}$$

[32] Note that as long as $k_{t+1} \leq \widetilde{k}$, there is no expenditure on education in the economy as a whole. Hence, $H_{t+1} = 1$ and $k_{t+1} = K_{t+1}$.

Hence, it follows from (12) that

$$b_{t+1}^i = \max\left\{ \begin{array}{cc} \beta[w(k_{t+1})h(b_t^i) - \theta] & if \quad b_t^i \le e(k_{t+1}) \\ \\ \beta[w(k_{t+1})h\left(e(k_{t+1})\right) + \left(b_t^i - e(k_{t+1})\right) R(k_{t+1}) - \theta] & if \quad b_t^i > e(k_{t+1}) \end{array} , \ 0 \right\}. \quad (21)$$

Namely, intergenerational transfers within group $i$ in period $t+1$, $b_{t+1}^i$ are determined by the intergenerational transfers within the group in the proceeding period, as well as the rewards to factors of production, as determined by the capital-labor-ratio in the economy. i.e.,

$$b_{t+1}^i \equiv \phi(b_t^i, k_{t+1}). \quad (22)$$

Let $\widehat{k}$ be the critical level of the capital-labor ratio below which individuals who do not receive transfers from their parents (i.e., $b_t^i = 0$ and therefore $h(b_t^i) = 1$) do not transfer income to their offspring. That is, $w(\widehat{k}) = \theta$. As follows from (2), $\widehat{k} = [\theta/(1-\alpha)A]^{1/\alpha} \equiv \widehat{k}(\theta)$, where if $k_{t+1} \le \widehat{k}$ then $w(k_{t+1}) \le \theta$, whereas if $k_{t+1} > \widehat{k}$ then $w(k_{t+1}) > \theta$. Hence, intergenerational transfers within group $i$ in period $t+1$, $b_{t+1}^i$ is positive if and only if $k_{t+1} > \widehat{k}$, i.e.,

$$b_{t+1}^i = \phi(0, k_{t+1}) \left\{ \begin{array}{ll} = 0 & if \quad k_{t+1} \le \widehat{k}; \\ \\ > 0 & if \quad k_{t+1} > \widehat{k}. \end{array} \right. \quad (23)$$

In order to reduce the number of feasible scenarios for the evolution of the economy, suppose that once wages increase sufficiently such that members of group $P$ transfer resources to their offspring, i.e., $k_{t+1} > \widehat{k}$, investment in human capital is profitable, i.e., $k_{t+1} > \widetilde{k}$. That is,[33]

$$\widetilde{k} \le \widehat{k}. \quad \text{(A2)}$$

Let $\widetilde{t}+1$ be the first period in which the capital labor ratio exceeds $\widetilde{k}$ (i.e., $k_{\widetilde{t}+1} > \widetilde{k}$). That is, since $k_0 < \widetilde{k}$, it follows that $k_{t+1} \le \widetilde{k}$ for all $0 \le t < \widetilde{t}$. Let $\widehat{t}+1$ be the first period in which the capital labor ratio exceeds $\widehat{k}$. That is, $k_{t+1} \le \widehat{k}$ for all $0 \le t < \widehat{t}$. It follows from Assumption A2 that $\widetilde{t} \le \widehat{t}$.

The evolution of transfers within *each* of the two groups, as follows from the fact that $k_{t+1} = \kappa(b_t^R, b_t^P)$, is fully determined by the evolution of transfers within *both* types of dynasties. Namely,

$$b_{t+1}^i = \phi(b_t^i, k_{t+1}) = \phi(b_t^i, \kappa(b_t^R, b_t^P)) \equiv \psi^i(b_t^R, b_t^P); \qquad i = R, P, \quad (24)$$

where the initial transfers of the Capitalists and the Workers are

$$\begin{aligned} b_0^R &= \max[\beta[w(k_0) + k_0 R(k_0)/\lambda - \theta] \ , 0]; \\ b_0^P &= \max[\beta[w(k_0) - \theta] \ , 0], \end{aligned} \quad (25)$$

noting that the level of human capital of every adult $i$ in period 0 is $h_0^i = 1$, and the entire stock of capital in period 0 is distributed equally among the Capitalists.

[33]Since $\widehat{k} = \widehat{k}(\theta)$, where $\widehat{k}'(\theta) > 0$, it follows that for any given $\gamma$, there exists $\theta$ sufficiently large such that $\widetilde{k}(\gamma) \le \widehat{k}(\theta)$.

**Lemma 1** *The intergenerational transfers of members of group R (the Rich) is higher than that of members of group P (the poor) in every time period, i.e.,*

$$b_t^R \geq b_t^P \quad \textit{for all } t.$$

**Proof.** As follows from (22) $b_{t+1}^i$ is increasing in $b_t^i$. Hence, since (25) implies that $b_0^R \geq b_0^P$ it follows that $b_t^R \geq b_t^P$ for all $t$. □

## 3 The Process of Development

This section analyzes the endogenous evolution of the economy from early to mature stages of development. The dynamical system is uniquely determined by the joint-evolution of the intergenerational transfers of members of groups $P$ and $R$. As follows from (24), the evolution of the economy is given by the sequence $\{b_t^P, b_t^R\}_{t=0}^{\infty}$ that satisfies in every period

$$\begin{aligned} b_{t+1}^P &= \psi^P(b_t^R, b_t^P); \\ b_{t+1}^R &= \psi^R(b_t^R, b_t^P), \end{aligned} \tag{26}$$

where $b_0^P$ and $b_0^R$ are given by (25).

As will become apparent, if additional plausible restrictions are imposed on the basic model, the economy endogenously evolves through two fundamental regimes:

- Regime I: In this early stage of development the rate of return to human capital is lower than the rate of return to physical capital and the process of development is fueled by capital accumulation.

- Regime II: In these mature stages of development, the rate of return to human capital increases sufficiently so as to induce human capital accumulation, and the process of development is fueled by human capital as well as physical capital accumulation.

In Regime I, physical capital is scarce and the rate of return to human capital is therefore lower than the rate of return to physical capital. Since there is no incentive for investment in human capital the process of development is fueled by capital accumulation. The wage rate is lower than the critical level that would enable individuals who do not own any capital to engage in intergenerational transfers (and thus savings). The Poor, therefore, consume their entire wages, they are not engaged in saving, capital accumulation, and intergenerational transfers. Their decedents, therefore, are also unable to engage in savings and intergenerational transfers and the Poor are in a temporary steady state equilibrium in which there is neither investment in physical capital nor in human capital. In contrast, the income of the Rich, who own the entire stock of capital in the economy, is sufficiently high, permitting intergenerational transfers and capital accumulation. Intergenerational transfers among the Rich increase over time and the stock of physical capital in the economy, therefore, increases as well. During this regime, physical capital accumulation by the rich raises the wages and therefore the return to human capital and decreases the return to physical capital. However, as long as the rate of return to human capital remains lower than the rate of return to physical capital, the qualitative structure of the economy remains unchanged. That is, the Poor are in a poverty trap, the Rich get richer and the process of development is based solely on physical capital accumulation. Inequality in Regime I, increases the wealth of individuals whose marginal propensity to save in higher and consequently increases aggregate savings and capital accumulation and enhances the process of development.

The accumulation of physical capital by the Rich in Regime I raises gradually the rate of return to human capital. Ultimately, the rate of return to human capital is sufficiently high so as to induce human capital accumulation, and the economy enters into Regime II where the process of development is fueled by human capital accumulation as well as physical capital accumulation.

Regime II is subdivided into three stages. In Stage I, investment in human capital is selective and it is feasible only for the Rich. In Stage II, investment in human capital is universal but it is still sub-optimal due to binding credit constraints, and in Stage III, investment in human capital is optimal since credit constraints are no longer binding.

Stage I: (Selective Human Capital Accumulation): In this stage, the capital labor ratio in the economy is higher than that in Regime I, and although it generates wage rates that justify investment in human capital, these wages are still lower than the critical level that would permit intergenerational transfers for individuals who do not own any capital. Hence, although the rate of return justifies investment in human capital, in the absence of parental support, credit constraints deprives the Poor from this investment. The Poor consume their entire income and they are not engaged in saving and capital accumulation. Their decedents are therefore unable to engage in savings and intergenerational transfers and the Poor remain in a temporary steady state equilibrium in which there is neither investment in physical capital nor in human capital. In contrast, the income of the Rich is sufficiently high, permitting intergenerational transfers and physical capital accumulation as well as human capital accumulation. Intergenerational transfers and the accumulation of physical capital by the Rich gradually rise in Stage I of Regime II, and ultimately the wage rate is sufficiently high so as to permit some investment in human capital by the Poor (i.e., the economy enters stage II of Regime II).

Stage II (Universal Human Capital Accumulation): In this stage, the capital labor ratio in the economy generates wage rates that permit some invest in human capital by all individuals. In contrast to the Rich, the investment of the poor is constrained by parental wealth and it is therefore sub-optimal. That is, the marginal return on investment in human capital among the Poor is higher than that among the Rich. Equality alleviates the adverse effect of credit constraints on the investment of the Poor in human capital, and has therefore a positive effect on the level of human capital and economic growth. The gradual increase in the wage income of the decedents of the Poor that takes place in Stage II of Regime II, due to a gradual increase in their investment in human capital, makes the credit constraint less binding over time and the aggregate effect of income distribution on the growth process subsides.

Stage III (Unconstrained Investment in Human Capital). In Stage III, credit constraints are non-binding due to the increase in wage income in Stage II, the rate of return to human capital is equalized across groups, and inequality therefore has no effect on economic growth.

### 3.1 Regime I: Physical Capital Accumulation

In this early stage of development the rate of return to human capital is lower than the rate of return to physical capital and the process of development is fueled by capital accumulation. Regime I is defined as the time interval $0 \leq t < \widetilde{t}$. In this early stage of development the capital-labor ratio in period $t+1$, $k_{t+1}$, which determines the return to investment in human capital in period $t$, is lower than $\widetilde{k}$. The rate of return to human capital is therefore lower than the rate of return to physical capital, and the process of development is fueled by capital accumulation.[34] As follows from (11) the level of real expenditure on education in Regime I is therefore zero and members of both groups acquire only basic skills. That is, $h(e(k_{t+1})) = 1$. Furthermore, as established in the following lemma, since the income of members of group $P$ (the Poor) is lower than the threshold that permits intergenerational transfer there are no intergenerational transfers among dynasties of this group.

**Lemma 2** *Under Assumptions A1 and A2, in Regime I and Stage I of Regime II (i.e., for the time interval $0 \leq t \leq \widehat{t}$), there are no intergenerational transfers among dynsaties of group $P$ (the Poor), i.e.,*

$$b_t^P = 0 \quad for \quad 0 \leq t \leq \widehat{t}$$

**Proof.** As follows from the definition of $\widehat{k}$, if $k_t \leq \widehat{k}$ then $w(k_t) \leq \theta$. Hence, since $k_0 < \widehat{k}$ it follows from (25) that $b_0^P = \max[\beta[w(k_0) - \theta]\ ,0] = 0$. Furthermore, for $1 \leq t \leq \widehat{t}$, as long as $b_{t-1}^P = 0$ the descendents of members of group $P$ do not invest in human capital in period $t-1$, $h_t^P = 1$, and therefore $b_t^P = \max[\beta[w(k_t) - \theta]\ ,0] = 0$. □

[34] As argued, two assumptions assure that the return to physical capital is larger than the return to human capital in early stages of development: (1) The capital-labor ratio is low since the initial stock of capital is low and individuals can supply labor even if no real resources are invested in education, and (2) The slope of the production function of human capital is finite at the origin. (As is also the case for the production of physical capital).

As follows from (15)-(19), and Lemma 2, since $e_t^R = e_t^P = b_t^P = 0$ in the time interval $0 \leq t < \widetilde{t}$, (where $\widetilde{t} \leq \widehat{t}$ as follows from A2) the capital-labor ratio $k_{t+1}$, is determined in Regime I by the intergenerational transfers of members of group $R$, according to their fraction in the population $\lambda$; $k_{t+1} = \kappa(b_t^R, 0) = \lambda b_t^R$ for $0 \leq t < \widetilde{t}$ (i.e., for $k_{t+1} \in (0, \widetilde{k})$). Since $b_t^R \in [0, \widetilde{b}]$ for $0 \leq t < \widetilde{t}$,

$$k_{t+1} = \kappa(b_t^R, 0) = \lambda b_t^R \quad \text{for} \quad b_t^R \in [0, \widetilde{b}], \tag{27}$$

where $\widetilde{b} \equiv \widetilde{k}/\lambda = \alpha/[(1-\alpha)\gamma\lambda]$.[35]

**The Dynamics of Transfers**

**A. Unconditional Dynamics**

The evolution of the economy in Regime I, as follows from (26) and Lemma 2, is given by

$$\begin{aligned} b_{t+1}^R &= \psi^R(b_t^R, 0) = \max[\beta[w(\lambda b_t^R) + b_t^R R(\lambda b_t^R) - \theta], 0]; \\ b_{t+1}^P &= \psi^P(b_t^R, 0) = \max[\beta[w(\lambda b_t^R) - \theta], 0] = 0, \end{aligned} \tag{28}$$

for $b_t^R \in [0, \widetilde{b}]$, where $b_0^P = 0$ and $b_0^R$ is given by (25).

In order to assure that the economy would ultimately take off from Regime I to Regime II, it is assumed that the technology is sufficiently productive. That is,[36]

$$A \geq \underline{A} \equiv A(\alpha, \gamma, \lambda, \beta, \theta). \tag{A3}$$

As depicted in Figure 1 and established in Appendix 1, the function $\psi^R(b_t^R, 0)$ is equal to zero for $b_t^R \leq \underline{b}$, it is increasing and concave for $\underline{b} < b_t^R \leq \widetilde{b}$ and it crosses the $45^0$ line once in the interval $\underline{b} < b_t^R < \widetilde{b}$.

Hence, the dynamical system $\psi^R(b_t^R, 0)$, depicted in Figure 1, has two steady-state equilibria in the interval $b_t^R \in [0, \widetilde{b}]$; A locally stable steady-state, $\overline{b} = 0$, and an unstable steady-state, $\overline{b}^u \in (\underline{b}, \widetilde{b})$. If $b_t^R < \overline{b}^u$ then the transfers within each dynasty of type $R$ contract over time and the system converges to the steady-state equilibrium $\overline{b} = 0$. If $b_t^R > \overline{b}^u$ then the transfers within each dynasty of type $R$ expand over the entire interval $(\overline{b}^u, \widetilde{b}]$, crossing into Regime II. To assure that the process of development starts in Regime I and ultimately reaches Regime II, it is assumed that[37]

$$b_0^R \in (\overline{b}^u, \widetilde{b}). \tag{A4}$$

[35]Note that one can assure that the economy remains in Regime I for at least one period. For instance, since $k_0 \in (0, \widetilde{k}(\gamma))$ there exist a sufficiently large $\theta$ and a sufficiently small $\gamma$ such that the economy is in Regime I in period 0. In particular, as follows from Lemma 2, $b_0^R$ is decreasing in $\theta$ and is independent of $\gamma$. Furthermore, $\widetilde{k}$ is decreasing in $\gamma$ and $\widehat{k}$ is increasing in $\theta$. Hence, since $k_1 = \lambda b_0^R$ if $\lambda b_0^R \leq \widetilde{k}$ there exist a sufficiently small level of $\gamma$ and a sufficiently large level $\theta$ such that $k_1 \leq \widetilde{k}$ and the economy is in Regime I in period 0.

[36]The precise value of $\underline{A}$ is a cumbersome expression of these five parameters.

[37]As follows from (25), there exists a feasible set of parameters $A, \alpha, \beta, k_0, \theta$, and $\lambda$ that satisfy Assumptions A1-A3 such that $b_0^R \in (\overline{b}^u, \widetilde{b})$. In particular, given the initial level of capital, if the number of Capitalist in the initial period is sufficiently small $b_0^R > \overline{b}^u$.

**B. Conditional Dynamics**

In order to visualize the evolution of the threshold for the departure of members of group $P$ from the zero transfer state, the dynamics of transfers within dynasties is depicted in Figure 2(a), for a given $k$. This conditional dynamical system is given by (20). For a given $k \in (0, \widetilde{k}]$, the dynamic of transfers within dynasty $i$, is

$$b_{t+1}^{i} = \phi(b_t^i; k) = \max\{\beta[w(k) + b_t^i R(k) - \theta], 0\}. \tag{29}$$

Hence, there exist a critical level $b(k)$ below which $\phi(b_t^i; k) = 0$ and above which $\phi(b_t^i; k)$ is linear in $b_t^i$, with a slope $\beta R(k) > 1$, i.e.,

$$\begin{aligned} &\phi(b_t^i; k) = 0 && for \quad 0 \leq b_t^i \leq b(k); \\ &\partial\phi(b_t^i; k)/\partial b_t^i = \beta R(k) > 1 && for \quad b_t^i > b(k). \end{aligned} \tag{30}$$

Note that under Assumption A3 $\beta R(k) > 1$. Otherwise $\psi^R(b^R, 0) < b^R$ for $b^R \in (0, \widetilde{b}]$, in contradiction to Lemma 4.

As depicted in Figure 2(a), in Regime I, members of group $P$ are trapped in a zero transfer temporary steady-state equilibrium, whereas the level of transfers of members of group $R$ increases from generation to generation. As the transfers of members of group $R$ increase the capital-labor ratio increases and the threshold level of transfer, $b(k)$, that enables dynasties of type $P$ to escape the attraction of the no-transfer temporary steady-state equilibrium, eventually declines.

**Redistribution and the Dynamics of Output Per Worker**

The evolution of output per worker, $Y_t$, in Regime I, follows from (1),(2),(27) and (28). Provided that Assumption A4 is satisfied, output per worker, $Y_{t+1}$, is

$$Y_{t+1} = A\left[\beta\left\{\lambda[(1-\alpha)Y_t - \theta] + \alpha Y_t\right\}\right]^{\alpha} \equiv Y(Y_t), \tag{31}$$

where $Y'(Y_t) > 0$.

In order to examine the effect of inequality on economic growth, suppose that income in period $t$ is distributed differently between group $R$ and group $P$.[38] That is, the income of members of group $i$, $\check{I}_t^i$, is

$$\begin{aligned} &\check{I}_t^R = I_t^R - \varepsilon_t \equiv I^R(I_t^R, \varepsilon_t); \\ &\check{I}_t^P = I_t^P + \lambda\varepsilon_t/(1-\lambda) \equiv I^P(I_t^P, \varepsilon_t), \end{aligned} \tag{32}$$

where $\varepsilon_t$ is sufficiently small in absolute value such that: (i) the economy does not depart from its current stage of development, and (ii) the net income of members of group $P$ remains below that of

[38] Although one can view the change as a non-distortionary transfer from group $R$ to group $P$, we advocate a different interpretation. That is, a comparison between two hypothetical paths starting from different initial conditions in a given stage of development.

member of group $R$. The transfer of member $i$ of generation $t$ to the offspring is therefore

$$b_t^i = \max\{\beta[I^i(I_t^i, \varepsilon_t) - \theta], 0\} \equiv b^i(I_t^i, \varepsilon_t) \quad i = P, R. \tag{33}$$

**Proposition 1** *(The effect of inequality on economic growth in Regime I). Under Assumptions A2-A4, in every period in which income is redistributed less equally (between groups) the growth rate of output per worker increases and output per worker increases in all subsequent periods.*

**Proof.** As long as the economy is in Regime I, $I^P(I_t^P, \varepsilon_t) < \theta$, and $\beta[I^R(I_t^R, \varepsilon_t) - \theta] \in (\overline{b}^u, \widetilde{b})$. Hence, it follows from (33) that $\partial b_t^P / \partial \varepsilon_t = 0$ and $\partial b_t^R / \partial \varepsilon_t < 0$. Hence $Y_{t+1} = A[\lambda b_t^R]^\alpha = A\{\lambda\beta[I^R(I_t^R, \varepsilon_t) - \theta]\}^\alpha$ declines in $\varepsilon_t$ and the growth rate of $Y_t$ increases if income is redistributed less equally (i.e., $\varepsilon_t < 0$). Moreover, as follows from (31), $Y_{t+2}$ increases in $Y_{t+1}$ and output increases in all the subsequent periods of Regime I. □

Inequality enhances the process development in Regime I since a transfer of wealth from members of group $R$ (the Rich) to members of group $P$ (the Poor - who do not save in this stage) would increase aggregate consumption, decrease aggregate intergenerational transfers, and thus would slow capital accumulation and the process of development. Further, educational subsidy would not promote the growth process since the return to physical capital exceeds the return to human capital.

**Remark 1** *If income is redistributed less equally within groups (i.e., if additional income groups are created), then redistribution would not affect output per-worker as long as the marginal propensity to save remains equal among all sub-groups of each of the original groups (i.e., $\beta$ for group $R$ and $0$ for group $P$). Otherwise, since saving is a convex function of wealth, inequality would promote economic growth.*

### 3.2 Regime II: Human Capital Accumulation

In these mature stages of development, the rate of return to human capital increases sufficiently so as to induce human capital accumulation, and the process of development is fueled by human capital as well as physical capital accumulation. In stages I and II members of group $P$ are credit constrained and their marginal rate of return to investment in human capital is higher than that on physical capital, whereas those marginal rates of returns are equal for members of group $R$ who are not credit constrained. In stage III all individuals are not credit constrained and the marginal rate of return to investment in human capital is equal to the marginal rate of return on investment in physical capital.

### 3.2.1 Stage I: Selective Human Capital Accumulation

Stage I of Regime II is defined as the time interval $\widetilde{t} \leq t \leq \widehat{t}$. In this time interval $k_{t+1} \in (\widetilde{k}, \widehat{k})$ and the marginal rate of return on investment in human capital is higher than the rate of return on investment in physical capital for individuals who are credit constrained (members of group $P$), whereas those rates of returns are equal for members of group $R$.[39]

As follows from (11) and Lemma 2, $e_t^R > 0$ and $e_t^P = 0$. Hence, given (18), it follows that the capital albor ratio, $k_{t+1}$ in the interval $k_{t+1} \in (\widetilde{k}, \widehat{k})$ is determined by the savings of members of group $R$, as well as their inestment in hmuan capital. Namely,

$$k_{t+1} = \frac{\lambda(b_t^R - e(k_{t+1}))}{1 - \lambda + \lambda h(e(k_{t+1}))}. \tag{34}$$

Since $e'(k_{t+1}) > 0$, it follows that $k_{t+1} = \kappa(b_t^R, 0)$ where $\partial\kappa(b_t^R, 0)/\partial b_t^R > 0$. Hence, there exist a unique value $\widehat{b}$ of the level of $b_t^R$ such that $k_{t+1} = \widehat{k}$. That is, $\kappa(\widehat{b}, 0) = \widehat{k}$..

**The Dynamics of Transfers**

**A. Unconditional dynamics**

The evolution of the economy in Stage I of Regime II, as follows from (24) and (26) is given by[40]

$$\begin{aligned} b_{t+1}^R &= \psi^R(b_t^R; 0) = \beta[w(k_{t+1})h(e(k_{t+1})) + (b_t^R - e(k_{t+1}))R(k_{t+1}) - \theta]; \\ b_{t+1}^P &= \psi^P(b_t^R; 0) = 0, \end{aligned} \tag{35}$$

for $b_t^R \in [\widetilde{b}, \widehat{b}]$.

In order the assure that the process of development does not come to a halt in this pre-mature stage of development (i.e., in order to assure that there is no steady-state equilibrium in stage I of Regime II) it is sufficient that $\beta[w(\lambda\widehat{b}) + \widehat{b}R(\lambda\widehat{b}) - \theta] > \widehat{b}$ - a condition that is satisfied under Assumption A3.[41] This condition assures that if the equation of motion in Regime I would remain in place in Stage I of Regime II, then there is no steady-state in Stage I. As established in Appendix 2 this condition is sufficient to assure that given the actual equation of motion in Stage I of Regime II, the system has no steady-state in this Stage.

Figure 1 depicts the properties of $\psi^R(b_t^R, 0)$ over the interval $b_t^R \in [\widetilde{b}, \widehat{b}]$. The transfers within each dynasty of type $R$ expand over the entire interval crossing into Stage II.

[39] In all stages of development members of group $R$ are not credit constrained. That is, $e_t < b_t^R$, and the level of investment in human capital, $e_t$, permits therefore a strictly positive investment in physical capital, $b_t^R - e_t$, by the members of group $R$. If $e_t \geq b_t^R$ and hence, as follows from Lemma 1, $e_t > b_t^P$ there would be no investment in physical capital, the return to investment in human capital would be zero and $e_t = 0 < b_t^R$ in contradiction to $e_t > b_t^R$.

[40] $b_{t+1}^R > 0$ in this interval since as established in Lemma 4 $b_t^R > 0$, and as follows from Lemma 5 $\partial\psi^R(b_t^R, 0)/\partial b_t^R > 0$.

[41] For any given $b > \widetilde{b}$, (where $\widetilde{b}$ is independent of $A$) since $\beta[w(\lambda b) + bR(\lambda b) - \theta]$ is strictly increasing in $A$, there exists a sufficiently large $A$ such that $\beta[w(\lambda b) + bR(\lambda b) - \theta] > b$. Note that $\widehat{b}$ decreases with $A$, however a sufficiently large $\theta$ assures that $\widehat{k} > \widetilde{k}$.

**B. Conditional dynamics**

In order to visualize the evolution of the threshold for the departure of dynasties of type $P$ from the zero transfer state, the dynamics of transfers within dynasties is depicted in Figure 2(b) for a given $k$. This conditional dynamical system is given by (22). For a given $k \in (\widetilde{k}, \widehat{k}]$

$$\begin{aligned} b^i_{t+1} &= \max \left\{ \begin{array}{ll} \beta[w(k)h(b^i_t) - \theta] & if \quad b^i_t \leq e(k) \\ \beta[w(k)h\left(e(k)\right) + \left(b^i_t - e(k)\right) R(k) - \theta] & if \quad b^i_t > e(k) \end{array}, \ 0 \right\} \\ &\equiv \phi(b^i_t, k). \end{aligned} \tag{36}$$

Hence, for a given $k \in (\widetilde{k}, \widehat{k})$ there exist a critical level $b(k)$ below which $\phi(b^i_t; k) = 0$ and above which $\phi(b^i_t; k)$ is increasing and concave in $b^i_t$. In particular, [42]

$$\begin{array}{lll} \partial\phi(b^i_t; k)/\partial b^i_t > \beta R(k) > 0 & for & b(k) < b^i_t < e(k); \\ \partial^2\phi(b^i_t; k)/\partial b^{i2}_t < 0 & for & b(k) < b^i_t < e(k); \\ \partial\phi(b^i_t; k)/\partial b^i_t = \beta R(k) > 1 & for & b^i_t \geq e(k). \end{array} \tag{37}$$

Note that $\phi(b^i_t, k) > b^i_t$ for all $b^i > \widetilde{b}$.

As depicted in Figure 2($b$), in Stage I of Regime II, members of group $P$ are still trapped in a zero transfer temporary steady-state equilibrium, whereas the level of transfers of members of group $R$ increases from generation to generation. As the transfer of members of group $R$ increases the capital-labor ratio increases and the threshold level of transfer, $b(k)$, that enables members of group $P$ to escape the attraction of the no-transfer temporary steady-state equilibrium, eventually declines and ultimately vanishes as the economy enter stage III..

Stage I of Regime II is an intermediate stage in which inequality has an ambiguous effect on the rate of economic growth. A transfer of wealth from members of group $R$ to some members of group $P$ that would leave the wealth of these individuals below the threshold $\theta$ would increase aggregate consumption, decrease aggregate intergenerational transfers, and thus would slow physical and human capital accumulation and the process of development. However a transfer from members of group $R$ to some members of group $P$ that would place the wealth of these individuals above the threshold $\theta$, would generate investment in human capital among these individuals, bringing about an increase in the aggregate stock of human capital that can offset the negative effect of the transfer on the accumulation of physical capital. Furthermore, redistribution of income from the Rich to the Poor would be more effective via education subsidy. There exists a level of financial transfers that would be harmful for the growth process (since a fraction of it would be consumed), whereas a similar level of educational subsidy would enhance the growth process.

[42] Note that the condition $\beta[w(\lambda\widehat{b}) + \widehat{b}R(\lambda\widehat{b}) - \theta] > \widehat{b}$ that follows from Assumption A3 and assures that there is no steady-state in Stage I of Regime II, implies that $\beta R(\widehat{k}) \geq 1$.

### 3.2.2 Stage II: Universal Human Capital Investment

Stage II of Regime II is defined as the time interval $\widehat{t} < t < t^*$, where $t^*$ is the time period in which the credit constraints are no longer binding for members of group $P$, i.e., $b^P_{t^*} \geq e_{t^*}$. In this time interval, the marginal rate of return on investment in human capital is higher than the marginal rate of return on investment in physical capital for members of group $P$, whereas these rates of return are equal for members of group $R$. As established previously once $t > \widehat{t}$ the economy exits Stage I of Regime II and enters Stage II of Regime II. In the initial period $k_{\widehat{t}+1} > \widehat{k}$ and therefore $b^P_{\widehat{t}+1} > 0$ and consequently as established in Appendix 3, the sequence $\{b^R_t, b^P_t\}$ increases monotonically over the time interval $\widehat{t} < t < t^*$.

As follows from (11), (12), and (18), in Stage II $e^P_t = b^P_t < e_t$ and $e^R_t = e_t$ and therefore the capital albor ratio is determined by intergenerational transfers and investment in human capital of both types of individuals.

$$k_{t+1} = \frac{\lambda(b^R_t - e(k_{t+1}))}{(1-\lambda)h(b^P_t) + \lambda h(e(k_{t+1}))}. \tag{38a}$$

Since $e'(k_{t+1}) > 0$, it follows that $k_{t+1} = \kappa(b^R_t, b^P_t)$ where $\partial\kappa(b^R_t, b^P_t)/\partial b^R_t > 0$ and $\partial\kappa(b^R_t, b^P_t)/\partial b^P_t < 0$..

**The Dynamics of Transfers**

**A. Unconditional dynamics**

The evolution of the economy, in Stage II of Regime II, (i.e., as long as credit constraints are still binding - $b^P_t < e_t$), as follows from (20) and (26), is given by

$$\begin{aligned} b^R_{t+1} &= \psi^R(b^R_t, b^P_t) = \beta[w(k_{t+1})h(e(k_{t+1})) + (b^R_t - e(k_{t+1}))R(k_{t+1}) - \theta]; \\ b^P_{t+1} &= \psi^P(b^R_t, b^P_t) = \max\{\beta[w(k_{t+1})h(b^P_t) - \theta], 0\}, \end{aligned} \tag{39}$$

where $k_{t+1} = \kappa(b^R_t, b^P_t)$.

The unconditional dynamical system in Stage II of Regime II is rather complex and the a sequence of technical results that are presented in Appendix 3 characterizes the properties of the system In particular, it is shown that intergenerational transfers within the two groups, $(b^R_t, b^P_t)$, increase monotonically over time in Stage II of Regime II and the economy necessarily enters into stage III of Regime II.

**B. Conditional dynamics**

The evolution of transfers within dynasties is depicted in Figure 2(c) for a given $k > \widehat{k}$.[43] This

[43] Note that $k_t$ in stage II of Regime II may decline below $\widehat{k}$. In this case, conditional dynamics are described by (37). However, $b^P_t$ is non-decreasing in stage II of Regime II, that is, $b^P_t$ is above the threshold level $b = \phi(b, k)$ of (37).

conditional dynamical system is given by (22). For a given $k > \widehat{k}$,

$$b_{t+1}^i = \left\{ \begin{array}{ll} \beta[w(k)h(b_t^i) - \theta] & if \quad b_t^i \leq e(k) \\ \beta[w(k)h\left(e(k)\right) + \left(b_t^i - e(k)\right)R(k) - \theta] & if \quad b_t^i > e(k) \end{array} \right\} \equiv \phi(b_t^i, k). \tag{40}$$

Hence, for a given $k > \widehat{k}$, over the interval $0 < b_t^i < e(k)$, $\phi(b_t^i; k)$ is a positive, increasing, and concave function of $b_t^i$, where

$$\begin{array}{ll} \partial\phi(b_t^i; k)/\partial b_t^i > \beta R(k) > 0 & for \quad 0 < b_t^i < e(k); \\ \partial\phi(b_t^i; k)/\partial b_t^i = \beta R(k) & for \quad b_t^i \geq e(k). \end{array} \tag{41}$$

Note that for $k > \widehat{k}$ it follows that $\phi(b_t^i, k) > b_t^i$ for at least a strictly positive range $b_t^i \in [0, b]$, where $b > \widehat{b}$.

As depicted in Figure 2(c), in Stage II of Regime II, members of group $P$ depart from the zero transfer temporary equilibrium. The level of transfers of members of group $P$ increases from generation to generation. Eventually members of group $P$ are not credit constrained, i.e., $b_t^P \geq e_t$ and the economy endogenously enters into stage III of Regime II.

**Redistribution and the Dynamics of Output Per Worker**

Since in stage II and III of Regime II the income of each individual is greater than $\theta$, it follows from (13) that the marginal propensity to transfer is equal to $\beta$ among all individuals. The aggregate transfers of members of generation $t$, $\lambda b_t^R + (1-\lambda)b_t^P$, is therefore simply a fraction $\beta$ of $Y_t - \theta > 0$. That is,

$$\lambda b_t^R + (1-\lambda)b_t^P = \beta(Y_t - \theta). \tag{42}$$

The evolution of output per worker, $Y_t$, in Stage II of Regime II, as follows from (1),(15),(16), noting that $e_t^R = e_t$ and $e_t^P = b_t^P$, is therefore

$$Y_{t+1} = AK_{t+1}^{\alpha}H_{t+1}^{1-\alpha} = A[\beta(Y_t - \theta) - \lambda e_t - (1-\lambda)b_t^P]^{\alpha}[\lambda h(e_t) + (1-\lambda)h(b_t^P)]^{1-\alpha}. \tag{43}$$

Since $e_t = \arg\max\ [w_{t+1}h(e_t) - R_{t+1}e_t] = \arg\max\ Y_{t+1}$ (and since therefore $\partial Y_{t+1}/\partial e_t = 0$), it follows that

$$Y_{t+1} \equiv Y(Y_t, b_t^P), \tag{44}$$

where $\partial Y(Y_t, b_t^P)/\partial Y_t > 0$ and $\partial Y(Y_t, b_t^P)/\partial b_t^P > 0$, noting that as follows from (2) and (10), $h'(b_t^P) > h'(e_t) = \alpha/[(1-\alpha)k_{t+1}]$.

**Lemma 3** *Under A2-A4, $Y_t$ increases monotonically over Stage II.*

**Proof.** Follows from (42) and Corollary 3 (in Appendix 3). □

**Proposition 2** *(The effect of inequality on economic growth in Stage II of Regime II.) Suppose that income would have been distributed differently in Stage II of Regime II. Under Assumptions A2-A4, in every period in which income is redistributed more equally between groups the growth rate of output per worker increases and output per worker increases in all subsequent periods.*

**Proof.** As long as redistribution is sufficiently small in absolute value such that the economy remains in Stage II of Regime II (i.e., $I^P(I_t^P, \varepsilon_t) > \theta$ and $\beta[I^P(I_t^P, \varepsilon_t) - \theta] < e_t$) it follows from (33) that $\partial b_t^P / \partial \varepsilon_t > 0$ and $\partial b_t^R / \partial \varepsilon_t < 0$. Hence, as follows from the properties of the function in (44)

$$\frac{\partial Y_{t+1}}{\partial \varepsilon_t} = \frac{\partial Y(Y_t, b_t^P)}{\partial b_t^P} \frac{\partial b_t^P}{\partial \varepsilon_t} > 0, \tag{45}$$

and therefore

$$\frac{\partial Y_{t+2}}{\partial \varepsilon_t} = \frac{\partial Y_{t+2}}{\partial b_{t+1}^P} \frac{\partial b_{t+1}^P}{\partial b_t^P} \frac{\partial b_t^P}{\partial \varepsilon_t} + \frac{\partial Y_{t+2}}{\partial Y_{t+1}} \frac{\partial Y_{t+1}}{\partial \varepsilon_t} > 0. \tag{46}$$

Hence, $\partial Y_{t+j} / \partial \varepsilon_t > 0$ for $j = 1, 2, 3, 4, ...$, and the Proposition follows. □

Inequality negatively affects the process development in Stage II of Regime II. A transfer of wealth from members of group $R$ to members of group $P$ would not affect aggregate consumption, and aggregate intergenerational transfers, but due to liquidity constraints would allow for a more efficient allocation of aggregate investment between physical and human capital.[44]

**Remark 2** *If income is redistributed less equally within groups then redistribution would not affect the aggregate level of intergenerational transfers as long as the marginal propensity to transfer, $\beta$, is equal among all member of the economy . However, redistribution of income among members of group $P$ implies a less efficient allocation of human capital due to the liquidity constraints and the concavity of $h(e_t^P)$. Redistribution among members of group $R$, as long as all the members of sub-groups of $R$ remain unaffected by credit constraint, will not affect output. If however redistribution makes some members of sub-groups of $R$ credit constrained, less equal redistribution will decrease the growth rate of output per worker and the level of output per worker in all subsequent periods.*

### 3.2.3 Stage III - Unconstrained Investment in Human Capital

Stage III of Regime II is defined as $t \geq t^*$ where credit constraints are no longer binding (i.e.,. $b_t^R \geq b_t^P \geq e_t$). In this time interval the marginal rate of return on investment in human capital is equal to the marginal rate of return on investment in physical capital for all individuals.

As follows from (12), in stage III of Regime II $e_t^P = e_t^R = e_t$. Hence, given (18) and (42) it follows that $k_{t+1}$ is given by

$$k_{t+1} = \frac{\beta[Y_t - \theta] - e(k_{t+1})}{h(e(k_{t+1}))}. \tag{47}$$

[44]If ability contributes to the production of human capital and individuals differ in their ability, as long as offspring's ability is not highly correlated with parental income the qualitative results would not be altered.

Since $e'(k_{t+1}) > 0$, it follows that $k_{t+1} = k(Y_t)$ where $k'(Y_t) > 0$ and $\lim_{Y_t \to \infty} k_{t+1} = \infty$.

**The Dynamics of Transfers and Output Per Worker**

The evolution of the economy in stage III of Regime II, as follows from (24) and (26), is given by

$$\begin{aligned} b_{t+1}^R &= \psi^R(b_t^R, b_t^P) = \beta[w(k_{t+1})h(e(k_{t+1})) + (b_t^R - e(k_{t+1}))R(k_{t+1}) - \theta]; \\ b_{t+1}^P &= \psi^P(b_t^R, b_t^P) = \beta[w(k_{t+1})h(e(k_{t+1})) + (b_t^P - e(k_{t+1}))R(k_{t+1}) - \theta]. \end{aligned} \tag{48}$$

The evolution of output per worker, $Y_t$, in Stage III of Regime II, is independent of the distribution of intergenerational transfers. As follows from (1) and (42)

$$Y_{t+1} = A[\beta(Y_t - \theta) - e_t]^{\alpha}[h(e_t)]^{1-\alpha}. \tag{49}$$

Since $e_t = \arg\max Y_{t+1}$, it follows that $\partial Y_{t+1}/\partial e_t = 0$ and therefore

$$Y_{t+1} = Y^{III}(Y_t), \tag{50}$$

where $Y^{III\,\prime}(Y_t) = \beta\alpha A k_t^{\alpha-1} > 0$, $Y^{III\,\prime\prime}(Y_t) < 0$ and $\lim_{Y_t \to \infty} Y^{III\,\prime}(Y_t) = 0$ since $\lim_{Y_t \to \infty} k_{t+1} = \infty$.

As established in Appendix 4, in Stage III of Regime II, $Y_t$ increases monotonically and converges to a unique, *locally stable,* steady-state equilibrium $\overline{Y} > 0$, where intergenerational transfers are positive and equal across all individuals, i.e., $\bar{b}^P = \bar{b}^R > 0$.

**Redistribution and the Dynamics of Output Per Worker**

**Proposition 3** *(The effect of inequality on economic growth when credit constraints are no longer binding). Suppose that income would have been distributed differently in Stage III of Regime II. Redistribution has no effect on output and growth.*

**Proof.** Follows from the fact that $Y_{t+1}$ in (50) is independent of the distribution of output per worker in period $t$ between the two groups. □

Inequality has no effect on the growth process in Stage III of Regime II, since in the absence of credit constraints investment in human capital is optimal and since the marginal propensity to save is equal across individuals a redistribution of income would not affect aggregate investment.

## 4 Inequality and Development

**Theorem 1** *Under Assumption A1-A4*

*(a) In the early stage of development when the process of development is driven by capital accumulation, inequality raises the rate of growth of output per worker over the entire stage .*

(*b*) *In the mature stage of development when the process of development is driven by universal human capital accumulation and credit constraints are binding, equality raises the growth rate of output per worker over the entire stage.*

**Proof.** The Theorem is a corollary of Propositions 1 and 2 and Remarks 1 and 2. □

In the early stage of development inequality is conducive for economic development. In this stage the rate of return to human capital is lower than the rate of return to physical capital and the process of development is fueled by capital accumulation. Since capital accumulation is the prime engine of growth and since the marginal propensity to save is an increasing function of the individual's wealth, inequality increases aggregate savings and capital accumulation and enhances the process of development. Inequality enhances the process development in Regime I since a transfer of wealth from members of group $R$ to members of group $P$ (who do not save in this stage) would increase aggregate consumption, decrease aggregate intergenerational transfers, and thus would slow capital accumulation and the process of development.

In mature stages of development, the rate of return to human capital increases sufficiently so as to induce human capital accumulation, and the process of development is fueled by human capital as well as physical capital accumulation. Since human capital is embodied in individuals and each individual's investment is subjected to diminishing marginal returns, the aggregate return to investment in human capital is maximized if the marginal returns are equalized across individuals. Equality therefore alliviates the adverse effect of credit constraints on investment in human capital and promotes economic growth.

## 5 Concluding Remarks

This paper presents a unified approach for the dynamic implications of income inequality on the process of development. The proposed theory provides an intertemporal reconciliation for conflicting viewpoints about the effect of inequality on economic growth. The paper argues that the replacement of physical capital accumulation by human capital accumulation as a prime engine of economic growth has changed the qualitative impact of inequality on the process of development. In early stages of industrialization as physical capital accumulation is a prime source of economic growth, inequality enhances the process of development by channeling resources towards the owners of capital whose marginal propensity to save is higher.[45] In later stages of development, however, as the return to human capital increases due to capital-skill complementarity, human capital becomes the prime

[45] In earlier stages of development, however, inequality that results in a larger share of income to the aristocrats may be harmful for the process of development, provided that their marginal propensity to consume is indeed high.

engine of growth. Since human capital is inherently embodied in humans and its accumulation is larger if it is shared by a larger segment of society, equality, in the presence of credit constraints, stimulates investment in human capital and promotes economic growth. As income increases, credit constraints gradually diminish and differences in saving rates decline, and the effect of inequality on economic growth becomes insignificant.[46]

The theory generates a testable implication about the effect of inequality on economic growth that may provide a greatly needed theoretical guidance to resolve the empirical controversy about this relationship. In contrast to the credit markets imperfection approach that suggests that the effect on inequality depends on the country's level of income (i.e., inequality is beneficial for poor economies and harmful for rich ones) the current research suggests that the effect of inequality on growth depends on the relative return to physical and human capital. As long as credit constraints are largely binding, the higher is the relative return to human capital the more harmful is inequality for economic growth. Hence, although the replacement of physical capital accumulation by human capital accumulation as a prime engine of economic growth in the currently developed economies is instrumental for the understanding of the role of inequality in their process of development,[47] the main insight of the paper is relevant for the currently less developed economies that have evolved differently. In the currently LDCs, the presence of international capital inflow diminishes the role of inequality in stimulating physical capital accumulation.[48] Moreover, the adoption of skilled-biased technologies, increases the return to human capital and thus, given credit constraints, strengthens the positive effect of equality on human capital accumulation and economic growth.

The incorporation of endogenous fertility decisions into the basic model would enrich the understanding of the reasons for the changing role of inequality in the process of development. If, for instance, individuals gain utility from the quantity and the wealth of their children, then as long as the income of poor families is insufficient to provide bequest for their children, poor individuals would choose high fertility rates that would negatively affect the capital labor ratio and hence offspring's income, delaying the timing of universal investment in human capital. However, once wages would increase sufficiently due to capital accumulation and the poor can afford bequeathing, there is an incentive to reduce the number of children, increasing the share of bequest to each child. The second phase of the transition to modern growth would be therefore accelerated.

[46]If heterogeneity in ability would be incorporated into the analysis, inequality at these mature stages of development may raise the incentives for investment and hence stimulates economic growth. See Galor and Tsiddon (1997), Hassler and Rodriguez-Mora (2000), Maoz and Moav (1999), Hassler et al. (2001).

[47]See Mokyr (1990, 1999).

[48]Furthermore, equality may contribute to economic growth in LDCs due to its beneficial effect on political stability, and fertility control.

The introduction of endogenous technological progress that is fueled by human capital accumulation would not affect the qualitative results. If human capital accumulation is conducive for economic growth, the optimal evolution of the economy would require the fastest capital accumulation in early stages of development so as to raise the incentive to invest in human capital. Inequality in early stages of development would therefore stimulate the process of development.

Finally, it is interesting to note that the effect of inequality on economic growth is qualitatively similar to the effect of assortative marriages on economic growth. In early stages of development since inequality is beneficial for growth, assortative marriages (i.e., sorting of couples by income) raise inequality and promote growth. However, in later stages of development in which equality contributes to economic growth, mixed marriages promote growth.

## Appendix 1

This appendix presents some technical results that are needed in order to characterize the dynamical system in Regime I.

**Lemma 4** *(The Properties of $\psi^R(b_t^R, 0)$). As depicted in Figure 1, under Assumptions A2 and A3, there exists $\underline{b} \in (0, \widetilde{b})$ such that $\psi^R(b_t^R, 0) = 0$ for $b_t^R \leq \underline{b}$. Furthermore, the function $\psi^R(b_t^R, 0)$ is increasing and strictly concave in the interval $b_t^R \in (\underline{b}, \widetilde{b}]$, and $\psi^R(\widetilde{b}, 0) > \widetilde{b}$.*

**Proof.** Follows from (2) and (28), noting that $\underline{b} = [\theta/A\lambda^\alpha(1 - \alpha + \alpha/\lambda)]^{1/\alpha}$ decreases in $A$ and $\widetilde{b} = \alpha/[(1 - \alpha)\lambda\gamma]$ is independent of $A$. □

**Corollary 1** *As depicted in Figure 1, under Assumptions A2 and A3, the dynamical system $\psi^R(b_t^R, 0)$ has two steady-state equilibria in the interval $b_t^R \in [0, \widetilde{b}]$; A locally stable steady-state, $\overline{b} = 0$, and an unstable steady-state, $\overline{b}^u \in (\underline{b}, \widetilde{b})$.*

## Appendix 2

This appendix presents some technical results that are needed in order to characterize the dynamical system in Stage I of Regime II.

**Lemma 5** *Under Assumptions A2 and A3, the properties of $\psi^R(b_t^R, 0)$ in the interval $b_t^R \in [\widetilde{b}, \widehat{b}]$ are*

$$\partial\psi^R(b_t^R, 0)/\partial b_t^R > 0$$

$$\psi^R(b_t^R, 0) > b_t^R$$

**Proof.** $\partial\psi^R(b_t^R,0)/\partial b_t^R > 0$ as follows from the properties of (2). Moreover, Lemma 4 and the condition $\beta[w(\lambda\widehat{b}) + \widehat{b}R(\lambda\widehat{b}) - \theta] > \widehat{b}$, imply that in the absence of investment in human capital $\beta[w(\lambda b_t^R) + b_t^R R(\lambda b_t^R) - \theta] > b_t^R$ for $b_t^R \in [\widetilde{b}, \widehat{b}]$. Since $\partial\psi^R(b_t^R,0)/\partial e_t^R > 0$ for $b_t^R \in (\widetilde{b}, \widehat{b}]$, and $e_t^R \in [0, e_t]$, it follows therefore that $\psi^R(b_t^R,0) \geq \beta[w(\lambda b_t^R) + b_t^R R(\lambda b_t^R) - \theta] > b_t^R$ for $b_t^R \in [\widetilde{b}, \widehat{b}]$. □

**Corollary 2** *The dynamical system $\psi^R(b_t^R,0)$ has no steady-state equilibria in the interval $b_t^R \in [\widetilde{b}, \widehat{b}]$.*

## Appendix 3

This appendix presents some technical results that are needed in order to characterize the dynamical system in Stage II of Regime II.

**Lemma 6** *Under Assumption A2-A4, $\partial\psi^i(b_t^R, b_t^P)/\partial b_t^j > 0$ for all $i, j = P, R$ in the time interval $\widehat{t} < t < t^*$.*

**Proof.** Follows from (1),(10), (38a) and (39), noting that (i) $h'(b_t^P) > \alpha/(1-\alpha)k_{t+1}$, and (ii) an increase in $b_t^P$ increases output per worker, and hence aggregate wage income, and decreases $e_t$. □

**Lemma 7** *Under Assumptions A2-A4, $b_t^P > 0$ in the time interval $\widehat{t} < t < t^*$.*

**Proof.** Given Lemma 5 and the definition of $\widehat{t}$, $b_{\widehat{t}+1}^R > b_{\widehat{t}}^R > 0$ and $b_{\widehat{t}+1}^P > b_{\widehat{t}}^P = 0$. Hence it follows from (39) and the positivity of $\partial\psi^i(b_t^R, b_t^P)/\partial b_t^j$ for all $i, j = P, R$, that $b_t^P > 0$ in the time interval $\widehat{t} < t < t^*$. □

**Lemma 8** *Under A2-A4, there exists no steady-state equilibrium in Stage II of Regime II.*

**Proof.** A steady-state equilibrium is a triplet $(k, b^P, b^R)$ such that $b^R = \phi(b^R, k)$, $b^P = \phi(b^P, k)$, and $k = \kappa(b^R, b^P)$. If there exist a non-trivial steady state in Stage II of Regime II then Lemma 1 and 7 implies that $(k, b^P, b^R) >> 0$. As follows from (30),(37) and (41), for any $k$ there exits at most one $b^i = \phi(b^i, k) > 0$. Hence, since $\phi$ is independent of $i = P, R$, if there exist a non-trivial steady-state then $b^P = b^R > 0$ and therefore $b_t^P > e_t$, and the steady-state is not in stage II of Regime II. □

**Corollary 3** *Under A2-A4, $(b_t^R, b_t^P)$ increases monotonically in Stage II of Regime II.*

**Proof.** Given Lemma 5 and the definition of $\widehat{t}$, $b_{\widehat{t}+1}^R > b_{\widehat{t}}^R > 0$ and $b_{\widehat{t}+1}^P > b_{\widehat{t}}^P = 0$. Hence since as follows from Lemma 6-8 $\partial\psi^i(b_t^R, b_t^P)/\partial b_t^j > 0$ for all $i, j = P, R$, and there exists no steady-state equilibrium in Stage II, $(b_t^R, b_t^P)$ increase monotonically in Stage II of Regime II. □

## Appendix 4

This appendix presents some technical results that are needed in order to characterize the dynamical system in Stage III of Regime II.

**Lemma 9** *Under A2-A4, $Y_t$ increases monotonically in Stage III of Regime II and converges to a unique, locally stable, steady-state equilibrium $\overline{Y} > 0$.*

**Proof.** As follows from the properties of the functions in (44), (49) and (50), $Y_{t+1} = Y^{III}(Y_t) = \max Y(Y_t, b_t^P)$. Hence, it follows from Lemma 3 that once the system enters Stage III $Y_{t+1} > Y_t$. Moreover, since $Y^{III}(Y_t)$ is strictly concave and since $\lim_{Y_t \to \infty} Y^{III\,\prime}(Y_t) = 0$, output increases monotonically converging to a unique, *locally stable,* steady-state equilibrium, $\overline{Y} > 0$. □

**Lemma 10** *Under A2-A4, the economy converges to a steady-state equilibrium where intergenerational transfers are positive and equal across all individuals, i.e.,*

$$\bar{b}^P = \bar{b}^R > 0.$$

**Proof.** As follows from the properties of (47) and Lemma 9 the economy converges to a unique steady-state level of the capital labor ratio, $\overline{k} = k(\overline{Y})$. As follows from (30),(37) and (41), given $\overline{k}$ it follows that $b^i = \overline{b}^i$ where $\overline{b}^i = \phi(\overline{b}^i, \overline{k})$, otherwise (since $\partial\phi(b^i, \overline{k})/\partial b^i \geq 0$) either [$b^i$ decreases (increases) for all $i$ and thus $k$ decreases (increases)] or [$b^R$ increases indefinitely and $b^P$ decreases to zero, and thus $k$ increases] in contradiction to the stationarity of $\overline{k}$. Hence, $\overline{b}^R = \phi(\overline{b}^R, \overline{k})$, $\overline{b}^P = \phi(\overline{b}^P, \overline{k})$, and $\overline{k} = \kappa(\overline{b}^R, \overline{b}^P)$. As follows from Lemma 4 and 5 there is no non-trivial steady-state equilibrium under which $b^P = 0$. Hence the steady-state equilibrium is $(\bar{b}^R, \bar{b}^P) >> 0$, where $\bar{b}^P = \bar{b}^R$ since $\phi$ is independent of $i = P, R$. □

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**Figure 1.** The dynamical system in Regime I and Stage I of Regime II

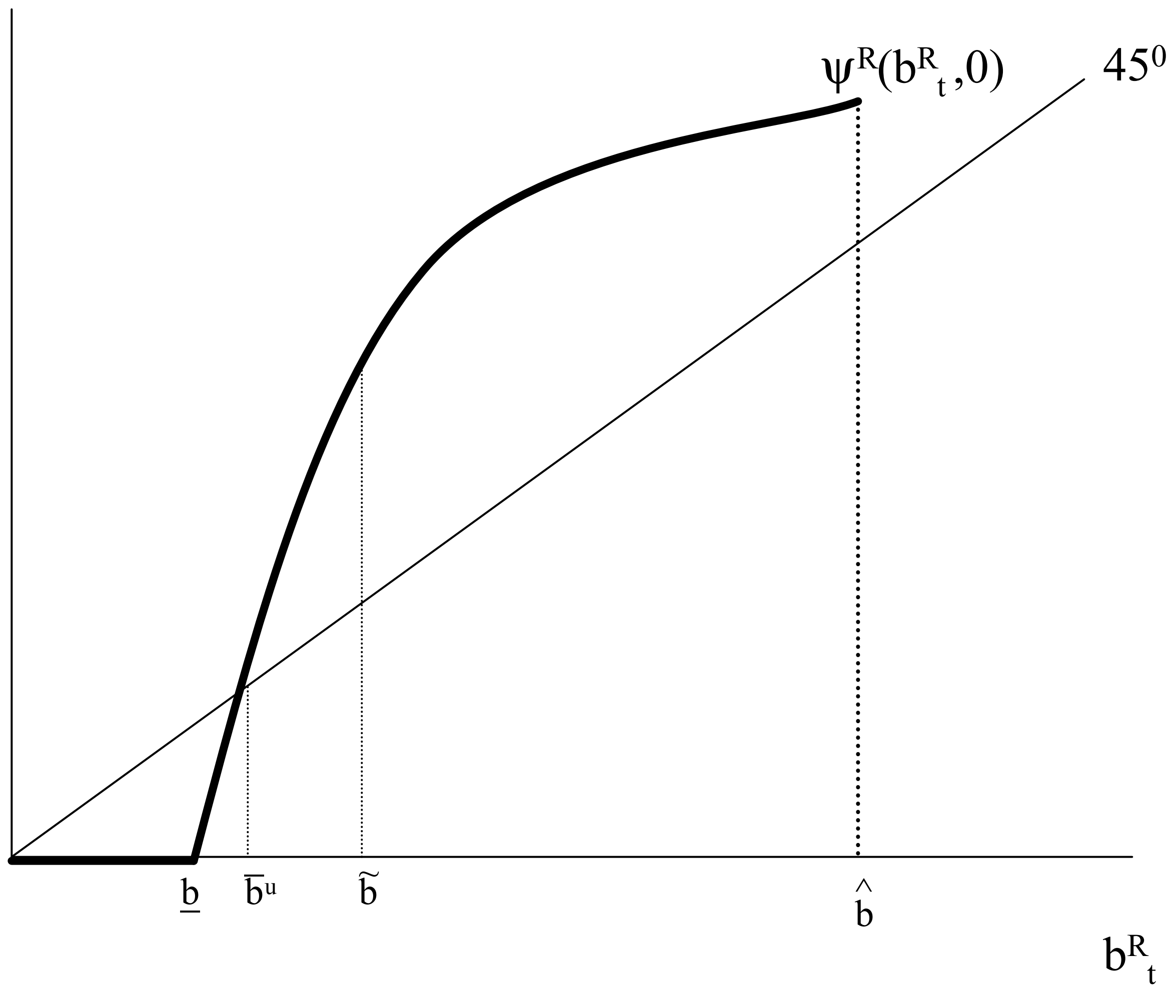

**Figure 2(a).** The conditional dynamical system in Regime I

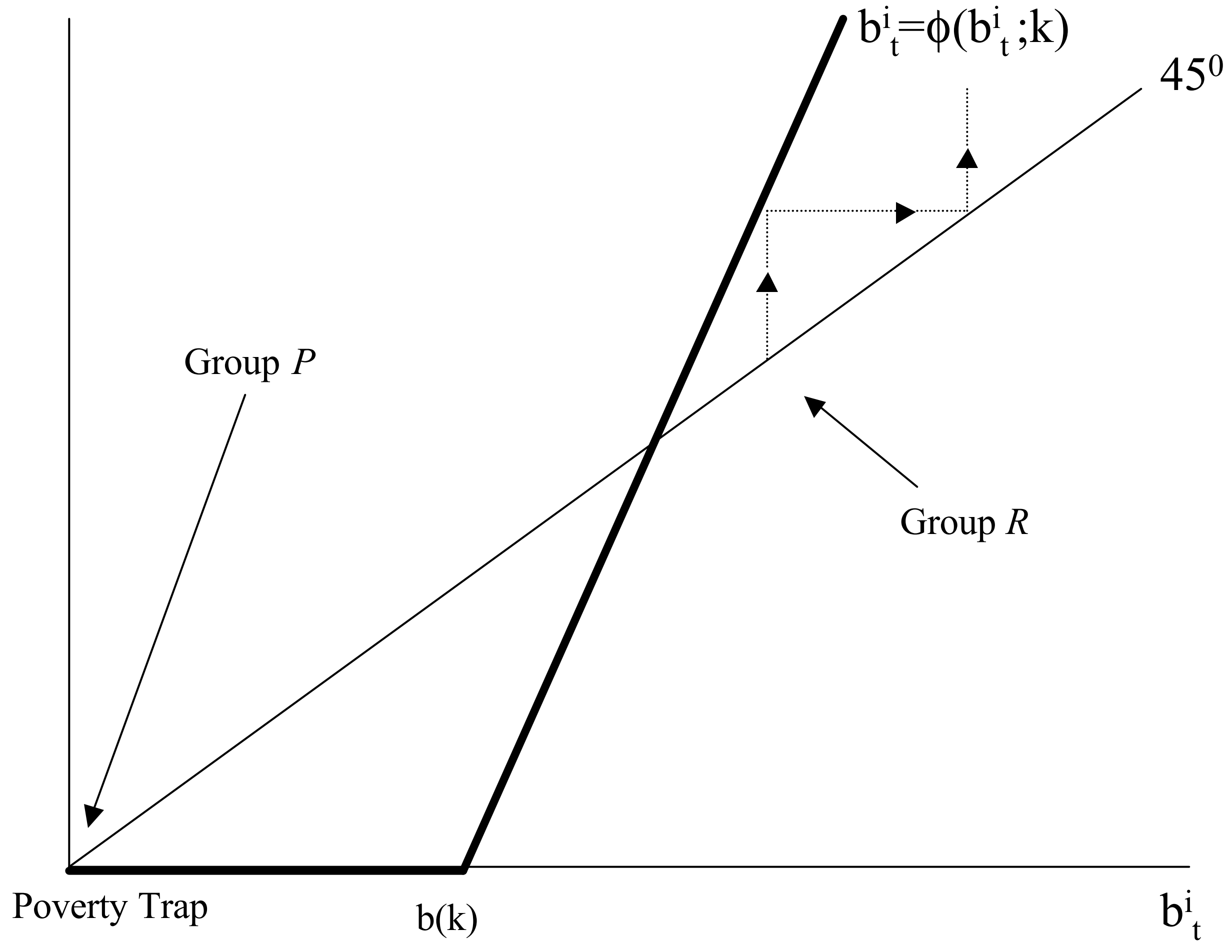

**Figure 2(b).** The conditional dynamical system in Stage I of Regime II

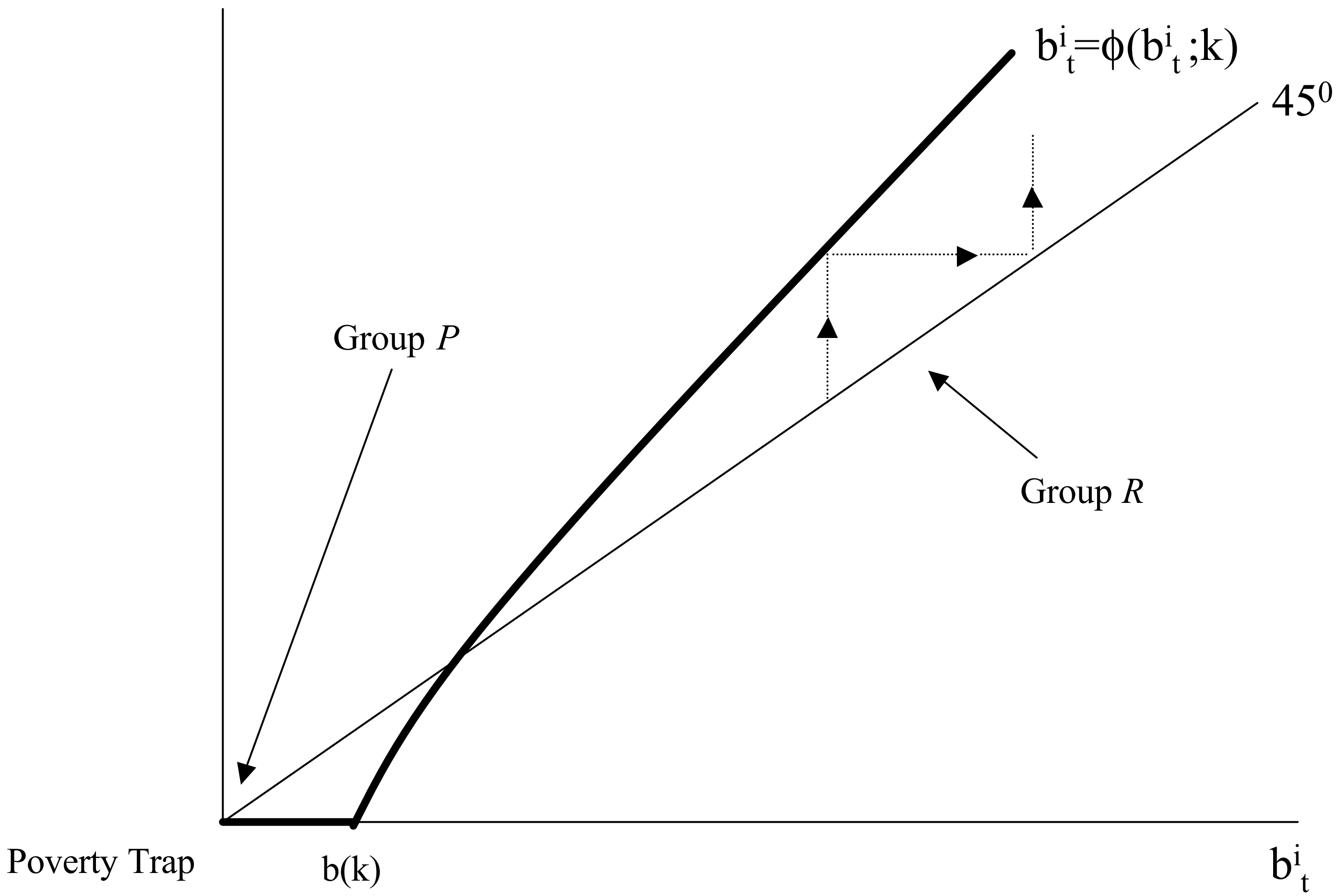

**Figure 2(c).** The conditional dynamical system in Stage II and III of Regime II